mass injection into the boundary layer and forced convection water cooling. In addition, a third configuration of having a buffer gas between the plasma and the wall can also be used to reduce the heat loss. However, considerable additional calculations would be needed for an engineering analysis of the proper amount and distribution of injection for efficient heat protection in a laser-heated thruster.

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Temperature Sensitivity and Erosive Burning

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REDUCTION of the effects of initial propellant temperature on the performance of solid rocket motors has been an elusive goal of propellant formulators for some time. Recently, nozzleless rocket motor tests have shown temperature sensitivities substantially below those for nozzled motors.[1] This suggests that paths to reduced motor temperature sensitivity may exist outside the propellant formulators domain. For nozzleless motors, the observed effects should be traceable to propellant deflection, erosive burning, and velocity profile effects. The purpose of this Note is to explore the effect of erosive burning on motor temperature sensitivity.

Erosive burning phenomena are complex and, currently, the cause is disputed. King[2] assumes that the primary effect is flame bending by the velocity gradient near the burning surface whereas Lengelle,[3] Beddini,[4] and Condon and Osborn[5] assume that the primary effect is due to eddy transport in the gas phase reaction zone. However, qualitative results from either school are similar. This is not unexpected. Turbulence near the burning surface is required to produce a steep velocity gradient there and Reynolds analogy is roughly valid for the combustion gases. For the purposes herein (scouting to see if the effect of erosion is beneficial, detrimental, or null) the simplest possible formulation consistent with the phenomena should be adequate. The Lenoir-Robillard relation[6] is simple, of closed form, contains the major physical elements, and has demonstrated its ability to correlate data. Unfortunately, a central aspect of that theory, the concept of erosion being dependent upon convective heat transfer from the "core gas" to the burning surface, has been shown to be fallacious by King.[2] However, this central dogma is more illusion than substance because of the interdependence of transport phenomena when the Prandtl and Schmidt numbers are close to unity. An energy balance at the burning surface gives

$$r\rho_s[c_s(T_s-T_i)-Q_s] \sim (\lambda_g+\overline{\rho_g c_g \epsilon})(T_f-T_s)/L \quad (1)$$

where r is the burning rate, ρ is density, c is specific heat, λ is thermal conductivity, T is temperature, Q_s is the energy release in the surface reaction, ϵ is turbulent diffusivity, and subscripts g, s, f, i, refer to gas, surface, flame, and initial conditions, respectively. For most composite solid propellants the activation energy of the surface reaction is large (O[30,000 kcal/mole]) so that T_s is substantially constant. Since,

$$r_o\rho_s[c_s(T_s-T_i)-Q_s] \sim \lambda_g(T_f-T_s)/L \quad (2)$$

for burning without crossflow and $r=r_o+r_e$

$$r_e\rho_s[c_s(T_s-T_i)-Q_s] \sim \overline{\rho_g c_g \epsilon}(T_f-T_s)/L \quad (3)$$

Lengelle[3] has shown that the characteristic thickness of the gas phase reaction zone L is independent of crossflow. Therefore, the variable in the turbulent problem is turbulent transport. For turbulent, convective heat transfer

$$h=\overline{\rho_g c_g \epsilon}/L \quad (4)$$

Thus, making use of turbulent heat transfer to evaluate the turbulent transport

$$r_e=h(T_f-T_s)/\{\rho_s[c_s(T_f-T_s)-Q_s]\} \quad (5)$$

Following Lenoir and Robillard

$$h=0.0288\, Gc_g Re_x^{-0.2} Pr^{-0.667}\exp(-\beta r\rho_s/G) \quad (6)$$

where G is the crossflow mass flux, Re_x is the length x Reynolds number, β is a parameter, and Pr is the Prandtl number.

Thus, the erosive burning rate r_e is

$$r_e=\alpha^*[(T_f-T_s)/\{\rho_s[c_s(T_s-T_i)-Q_s]\}]G^{0.8}x^{-0.2} \times\exp(-\beta r\rho_s/G) \quad (7)$$

where α^* is a parameter dependent upon gas phase fluid properties alone. Consequently, the Lenoir-Robillard relation is reached without heat transfer from a core flow. In short, Lenoir and Robillard have the "correct" equation but the wrong concept. The experimental work of Yamada et al.[7] also implies this.

To accomplish the stated task, burning rate is first expressed in terms of stationary and erosive terms. Then the erosive part is related to the crossflow. Finally, the crossflow is related to the static properties (p, M, T_i) defining the situation. Then by employing the proper definitions, the desired results may be achieved.

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With $r = r_o + r_e$

$$\partial \ell n r = \frac{\partial \ell n r_o + (r_e/r_o)\partial \ell n r_e}{1 + r_e/r_o} \tag{8}$$

From Eq. (7) with T_s, x, and Q_s constant

$$\partial \ell n r_e = \frac{\partial T_f}{T_f - T_s} + \frac{\partial T_i}{T^* - T_i} + \frac{(0.8 + \beta r \rho_s)}{G}\partial \ell n G - \frac{\beta r \rho_s}{G}\partial \ell n r \tag{9}$$

The mass flux $G = \rho_g u$, where u is crossflow speed. Therefore, for calorically perfect gases

$$\partial \ell n G = \partial \ell n p + \frac{1 + (\gamma - 1)M^2}{1 + (\gamma - 1)M^2/2}\partial \ell n M - \frac{\partial T_f}{(2T_f)} \tag{10}$$

where M is Mach number and γ is the specific heat ratio. Since the processes in the motor are substantially adiabatic,

$$\partial T_f = c_s \partial T_i / c_g \tag{11}$$

In quasisteady internal ballistic analyses of nozzled motors, port/throat geometries and, hence, the axial Mach number distribution are substantially independent of initial propellant temperature. Thus, for motor applications the most appropriate propellant sensitivities are those at constant Mach number. Therefore, by employing Eqs. (8-11)

$$\sigma_{p,M} = \left(\frac{\partial \ell n r_e}{\partial T_i}\right)_{p,M}$$

$$= \frac{\sigma_{p,o} + \dfrac{r_e}{r_o}\left[\dfrac{c_s/c_g}{T_f - T_s} + \dfrac{1}{T^* - T_i} - \dfrac{(0.8 + \beta r \rho_s/G)c_s}{2c_g T_f}\right]}{(1 + r_e/r_o)(1 + \beta r_e \rho_s/G)} \tag{12}$$

$$n_M = \left(\frac{\partial \ell n r}{\partial \ell n p}\right)_{T_i,M} = \frac{n_o + (r_e/r_o)(0.8 + \beta r \rho_s/G)}{(1 + r_e/r_o)(1 + \beta r_e \rho_s/G)} \tag{13}$$

For small port Mach numbers, total pressure losses are small and to an approximation (assuming $r = ap^n$)

$$\ell n p \sim (\ell n k_n + \ell n a + \ell n \rho_s + \ell n C^* - \ell n g_c)/(1 - n) \tag{14}$$

Since $C^* = \gamma R T_f / F(\gamma)$,[8] differentiation of Eq. (14) with respect to T_i (ρ_s, n assumed constant) yields

$$\pi_k = \left(\frac{\partial \ell n p}{\partial T_i}\right)_{k_n} \sim \frac{\sigma_p + c_s/(2c_g T_f)}{1 - n} \tag{15}$$

The effect of erosion on π_k should be approximated in this situation by employing the constant Mach number sensitivities because $M(x)$ is not strongly dependent upon T_i in a nozzled motor. Substituting Eqs. (12) and (13) into Eq. (15) yields

$$\pi_k \sim \frac{\sigma_{p,o} + \dfrac{c_s/c_g}{2T_f}\left[1 + \dfrac{r_e}{r_o}\left(\dfrac{2T_f}{T_f - T_s} + \dfrac{2T_f}{(c_s/c_g)(T^* - T_i)} + 0.2\right)\right]}{1 - n_o + 0.2\, r_e/r_o]} \tag{16}$$

For typical composite propellants[9] $O(T_f) \sim 3000°C$, $O(T_s) \sim 600°C$, $O(T^*) \sim 300°C$, $c_s \sim c_g/2$, and $O(T_i) \sim 25°C$. Therefore, retaining only the most significant terms of Eqs. (12) and (16) yields

$$\sigma_{p,M} \sim \frac{\sigma_{p,o} + (r_e/r_o)/(T^* - T_i)}{(1 + r_e/r_o)(1 + \beta r_e \rho_s/G)} \tag{17}$$

$$\pi_k \sim \frac{\sigma_{p,o} + (r_e/r_o)/(T^* - T_i)}{1 - n_o + 0.2 r_e/r_o} \tag{18}$$

With these relations and Eq. (13)

$$\sigma_{p,M} - \sigma_{p,o} \sim \frac{(r_e/r_o)[(T^* - T_i)^{-1} - \sigma_{p,o}(1 + \beta r \rho_s/G)]}{(1 + r_e/r_o)(1 + \beta r_e \rho_s/G)} \tag{19}$$

$$n_M - n_o \sim \frac{(r_e/r_o)[0.8 - n_o + (\beta r \rho_s/G)(1 - n_o)]}{(1 + r_e/r_o)(1 + \beta r_e \rho_s/G)} \tag{20}$$

$$\pi_k - \pi_{k,o} \sim \frac{(r_e/r_o)[(1 - n_o)/(T^* - T_i) - 0.2\,\sigma_{p,o}]}{(1 - n_o)(1 - n_o + 0.2\, r_e/r_o)} \tag{21}$$

Therefore:

1) If $(T^* - T_i)^{-1} > \sigma_{p,o}(1 + \beta r \rho_s/G)$, then erosion increases σ_p.

2) If $n_o \leq 0.8$, then erosion increases n.

3) If $(1 - n_o)/(T^* - T_i) > 0.2\ \sigma_{p,o}$, then erosion increases π_k.

For composite propellants at high pressure[9]

$$\sigma_{p,o} \sim 0.5/(T^* - T_i)$$

Therefore, erosive burning can reduce π_k only if $n_o > 0.9$. Moreover, erosive burning will reduce σ_p only if $\beta r \rho_s/G > 1$. Consequently, erosive burning will exert beneficial effects only for high burning rate propellants with very high exponents. For all other propellants, erosive burning can be expected to increase both temperature sensitivity and exponent.

In summary, the effect of erosive burning on temperature sensitivity and pressure exponent has been determined for small port Mach number situations by employing a modified Lenoir-Robillard relation. Results show that for typical composite propellants erosive burning increases both temperature sensitivity and pressure exponent.

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